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Numerical Simulation of Rip-Rap Revetments in Tidal Areas

Livia Mittelbach, Martin Pohl, Peter Schulze and Heinz Konietzky

Summary

Rip-rap revetments are used to protect embankments and coastal shores against erosion. They are built to resist ship and wind induced waves, tidal and ship induced currents, tidally varying water levels and storm surges. In some areas the current basis of rip-rap design is inadequate for dealing with the complexity and variety of boundary conditions, especially in tidal zones. A numerical model has therefore been developed which is capable of simulating the resistance of rip-rap to hydraulic loads. Rip-rap-water-interaction is modelled holistically using two numerical methods.

Rip-rap is modelled using the Discrete Element Method (DEM) in three dimensions. The DEM can be used to model rip-rap stones as autonomous objects with all degrees of freedom and realistic movement. The DEM code is coupled with a computational fluid dynamics code (CFD) to account for the influence of the hydraulic loads. Waves and currents acting on the rip-rap stones as well as tidally varying water levels can be generated realistically using time dependent boundary conditions.

Additional physical model tests in a laboratory flume, field tests and measurements with instrumented rip-rap stones serve as validation for the numerical model.

Keywords

revetment, rip-rap, numerical simulation, discrete element method, computational fluid dynamics, model tests

Zusammenfassung

Deckwerke dienen dem Schutz der Uferböschungen von Schiffahrtsstraßen vor auftretenden hydraulischen Belastungen aus schiffs- und windinduzierten Wellen und Strömungen, wechselnden Tidewasserständen sowie Sturmfluten. Die derzeitigen Grundlagen zur Deckwerksbemessung sind für die vielfältigen und komplexen Randbedingungen insbesondere im Tidegebiet teilweise ungenügend. Daher wird ein numerisches Modell entwickelt, mit dem die Interaktion zwischen Deckwerk und hydraulischen Belastungen simuliert werden kann. Zur ganzheitlichen Simulation werden dabei zwei verschiedene numerische Methoden verwendet.

Das Deckwerk wird mit der Diskreten Elemente Methode (DEM) modelliert. Das Verhalten der einzelnen Deckwerkssteine kann somit realistisch mit allen Freiheitsgraden abgebildet werden. Die DEM wird mit einem Programm zur numerischen Strömungssimulation (CFD) gekoppelt, um die hydraulischen Einwirkungen zu berücksichtigen.

Zusätzliche Modellversuche in einer hydraulischen Rinne, Feldversuche und Messungen mit instrumentierten Deckwerkssteinen dienen zur Validierung des numerischen Modells.

Schlagwörter

Deckwerk, Numerische Simulation, Diskrete Elemente Methode, Computational Fluid Dynamics, Modellversuche

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1 Introduction

Revetments are used to protect the banks of waterways and coastal shores against erosion. Because of the numerous advantages they offer, such as high flexibility and robustness against settlements, rip-rap revetments are used most frequently (Fig. 1). Revetments have to resist manifold influences. The main parameters affecting rip-rap design on inland waterways are ship-induced waves and currents as well as excess pore water pressures due to ship-induced water level drawdown. On waterways in coastal zones additional influences specific to the coast play an important role as well.



Figure 1: Rip-rap on the banks of the Ems River (Germany) as a protective layer to prevent erosion of bank slope by hydraulic loads.

The present guidelines used for rip-rap design in Germany are:

- BAW Code of Practice: “Principles for the Design of Bank and Bottom Protection for Inland Waterways” – GBB 2010 (BAW 2010)
- BAW Code of Practice: “Use of Standard Construction Methods for Bank and Bottom Protection on Inland Waterways” – MAR 2008 (BAW 2008)
- “Empfehlung für die Ausführung von Küstenschutzwerken” – EAK 2002 (KFKE 2007)
- “Wasserbausteine im Deckwerksbau” (HANSEN 1985)
- “The Rock Manual” (CIRIA et al. 2007)
- “Dikes and Revetments” / “Geosynthetics and Geosystems in Hydraulic and Coastal Engineering” (PILARCZYK 1998; PILARCZYK 2000)

The above criteria are mostly based on small-scale model tests or experience data from inland waterways. This means that, in some areas, these guidelines are of only limited applicability in view of the complex and diverse boundary conditions in tidal zones. Fig 2 gives an overview of the variety of influences which are relevant to rip-rap design for waterways in coastal areas. Larger, seagoing vessels with different geometries and increasing size instead of inland water vessels, different and irregular cross sectional areas of the waterway with varying slopes instead of a regular channel cross section, the influence of wind waves because of a larger, wind-exposed water surface, tidal currents in addition to ship-induced currents and tidal varying groundwater levels are just some of the factors that affect the different rip-rap design needed for inland and coastal waterways.

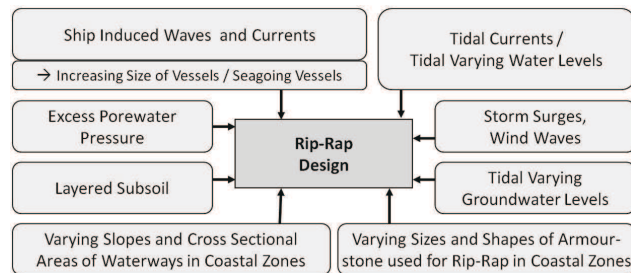


Figure 2: Influences on rip-rap design in tidal and coastal areas.

2 Research project

In the light of the insufficient basic criteria available for rip-rap design, especially in coastal zones, the aim is to develop a numerical model in 3D which is capable of simulating the rip-rap-water interaction. By coupling different numerical methods, the model takes the hydraulic part (waves and currents) as well as the mechanical part (the rip-rap) into consideration. Hence, a holistic numerical analysis of the stability of rip-rap revetments is possible.

The research project is divided into different parts: numerical simulation and model respectively field tests. The numerical modelling of the hydraulic part is done using a Computational Fluid Dynamics code (CFD), the simulation of the rip-rap is undertaken using the 3D Discrete Element Method (DEM). The holistic numerical modelling is carried out by a coupled computation of both codes. The numerical model is then validated

by model tests in a laboratory flume, measurement data from instrumented armourstones and measurement data of waves and currents from field tests.

The long-term objective of the research project is a suitable numerical tool for a safe and economic rip-rap design adapted to particular local conditions.

3 Design of rip-rap

The stability of rip-rap against hydraulic loads depends on the size and mass of the individual stones used as well as on the interaction of all the stones as an assembly. The interaction of all the stones in non-grouted rip-rap is achieved by means of interlocking effects between the armourstones. According to the BAW Code of Practice GBB (BAW 2010), the required individual stone size is determined in the hydraulic design of a revetment and depends on the acting hydraulic loads (waves, currents). The required mass per unit area (thickness of the rip-rap) is determined in the geotechnical design and is necessary to guarantee resistance to sliding failure, uplift and hydrodynamic soil displacement. Nonetheless, it is necessary for rip-rap to have a certain minimum thickness in order to ensure a stable armour layer with interlocking between the stones as referred to above. In addition to hydraulic and geotechnical design, the overall stability of the bank slope including the rip-rap is proved in the revetment design (BAW 2010).

Stone gradings according to the European standard for armourstone (DIN EN 13383-1) are used for rip-rap revetments. Light mass gradings (LMB_{5/40}, LMB_{10/60}) are mostly used on coastal waterways to produce appropriate and robust revetments.

In the numerical model the representation of the rip-rap should be realistic with regard to the particular armourstone grading category (size and mass of the stones) as well as the stability mechanism inside the rip-rap (interlocking effects).

4 Modelling of rip-rap with the DEM

The Discrete Element Method (DEM) is a numerical method which simulates the movement and interaction of particles of a discontinuous medium on the basis of Newton's second law of motion and a contact law. In a discontinuous medium, contacts or interfaces exist between the discrete bodies that make up the system (ITASCA 2014). An effective contact detection algorithm is therefore necessary in a DEM code in order to detect contacts, which are arising and breaking during the calculation progress, as well as corresponding contact laws that become active during interaction of particles (JAKOB and KONIETZKY 2012). The DEM was originally developed by CUNDALL and STACK (1979). Today it is widely used to examine engineering problems in granular and discontinuous materials.

The calculation cycle executed in the DEM is described in ITASCA (2014): "The calculations performed in the DEM alternate between the application of Newton's second law to the particles and a force-displacement law at the contacts. Newton's second law is used to determine the motion of each particle arising from the contact and body forces acting upon it, while the force-displacement law is used to update the contact forces arising from the relative motion at each contact." (Fig. 3)

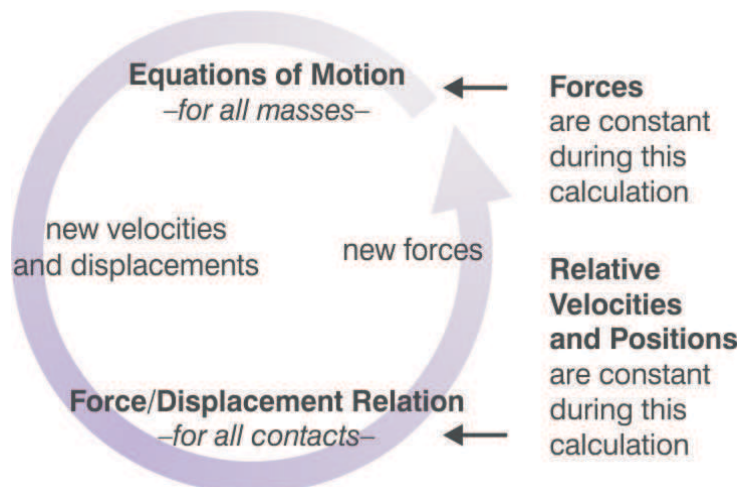


Figure 3: Calculation cycle DEM (ITASCA 2014).

The DEM allows rip-rap stones to be modelled as autonomous objects. The movement of stones with six degrees of freedom (three for translational and three for rotational movement) is represented realistically. In this research project the DEM part of the modelling is undertaken by using the three dimensional code PFC3D (Particle Flow Code 3D) developed by ITASCA CONSULTING GROUP INC. (2008a). This code is a simplified version of the general DEM because it utilises spherical particles (so-called balls) to make contact detection easier. However, arbitrary complex shapes can be produced by overlapping spheres (so-called clumps). This multi-sphere approach can be used to generate stone-like particles and the whole rip-rap (Fig. 4). Each clump acts as an independent object and cannot break during the calculation cycle (ITASCA 2008a).

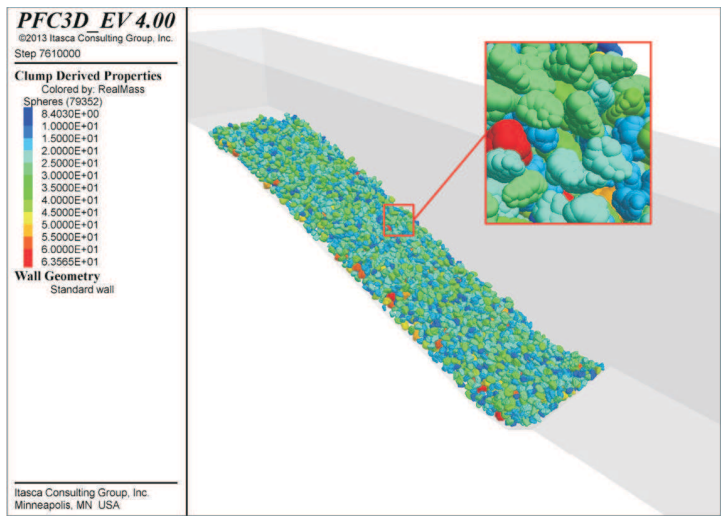


Figure 4: Rip-rap generated in PFC3D.

As referred to above the numerical rip-rap should be realistic with regard to the representation of interlocking effects between the stones as well as with regard to the particular armourstone grading category. This results in two important issues: the realistic representation of the particle shape of the individual stones and the realistic representation of the whole rip-rap with respect to the size and mass distribution of all stones in the numerical model.

The research project was carried out in cooperation with the Chair of Rock Mechanics and Rock Engineering, Geotechnical Institute, TU Bergakademie Freiberg, Germany. To reproduce the rip-rap in a realistic way, the stones were divided into different size and shape categories (platy, longish, compact) and a mixture of stones from all categories was used for the numerical model (HERBST et al. 2010). Several methods of generating irregular shaped particles and of representing rip-rap were also investigated, developed and tested (HERBST et al. 2010; YUAN 2012). The first method for the generation of rip-rap stones is a random algorithm developed on the basis of the procedure of LU and McDOWELL (2007). The shape of the generated particles depends on six different parameters: the number of directions for ball generation, the probability of each direction, the number of balls generated in the chosen direction, the degree of reduction of radii and the degree of overlapping of balls. Another method for the generation of rip-rap stones with realistic shapes is the use of surface meshes from real stones (obtained by photo or 3D-Scan). With the help of a bubble-pack-algorithm contained in the DEM code the 3D surface mesh is filled with balls. In both cases the rip-rap can be represented realistically regarding the size and mass distribution of the corresponding armourstone grading category (Fig. 5).

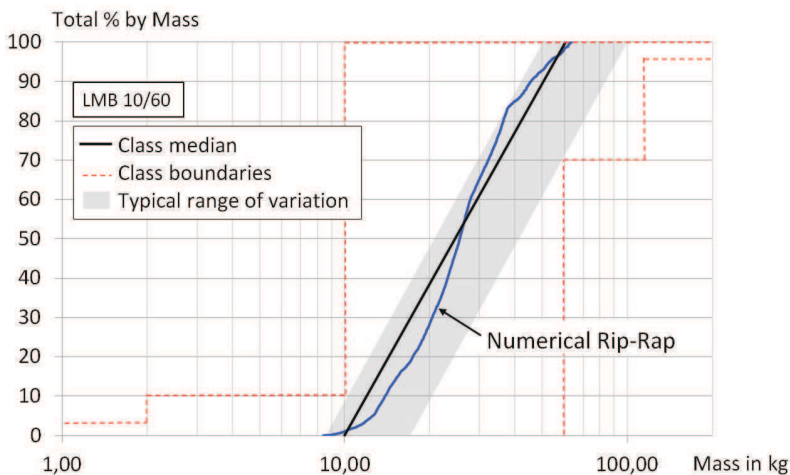


Figure 5: Typical range of variation of cumulative curve for armourstones class LMB_{10/60} (BAW 2008), cumulative curve of corresponding numerical rip-rap.

The stones in the numerical model can be represented in a detailed or simplified way (high or low resolution of particle shape), depending on the number of balls that are used to build one clump (Fig. 6). The more detailed the clumps are, the better the interaction of the stones is represented. However, the time required for the generation of the numerical rip-rap and the model calculation time in general increase exponentially if detailed

clumps are used. Besides the particle shape, the behaviour of the clumps during calculation is also affected by the friction coefficient on the particle surface and indirectly by the stiffness of the particles. The extent to which these properties affect the accuracy of the numerical modelling and the way the parameters should be chosen in order to produce the best match with reality is examined in physical model tests.

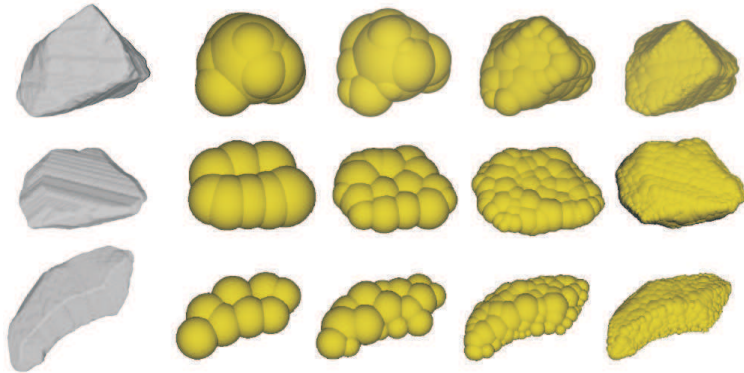


Figure 6: Representation of armourstones as clumps in simplified and detailed way (YUAN 2012).

5 Modelling of hydraulic influences with CFD

The modelling of the hydraulic part, such as waves and currents, is undertaken using a computational fluid dynamics (CFD) code. The hydrodynamic computation is done on the basis of the Navier-Stokes equations, which describe the motion of fluid substances. Together with the continuity equation they can be applied to solve all hydrodynamic problems.

In this research project the software “Coupled Computational Fluid Dynamics” (CCFD) is used, which is a product of ITOCHU Techno-Solution Corporation (ITASCA 2008b). CCFD solves the simplified incompressible Navier-Stokes equations (ITASCA 2008b, HERBST 2011):

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho \vec{v} \cdot \nabla (\vec{v}) = -\nabla p + \eta \nabla^2 (\vec{v}) \quad (1)$$

and the continuity equation (HERBST 2011):

$$\nabla \cdot \vec{v} = 0 \quad (2)$$

with ρ as the fluid density, \vec{v} as the fluid velocity, p as the fluid pressure and η as the fluid dynamic viscosity with the help of a finite volume method in a 3D discretized domain (fluid element, hexahedron). The coupled computation of both codes – the mechanical DEM calculation in PFC3D and the hydraulic computation in CCFD – is possible (Fig. 7). The CCFD-solver is embedded in a graphic modeller (pre/post-processor) which serves to specify the model geometry and the initial and boundary conditions (INTERNATIONAL CENTER FOR NUMERICAL METHODS IN ENGINEERING 2008).

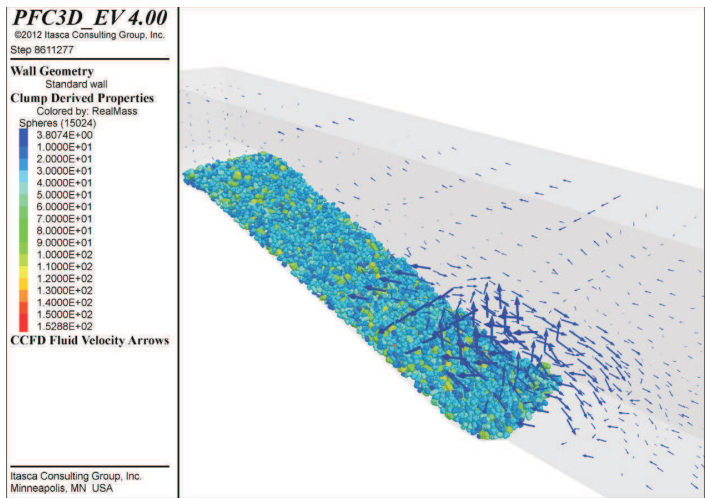


Figure 7: Coupled computation of wave impact on rip-rap.

In the numerical simulation the hydraulic loads can be generated by applying time-varying boundary conditions at the model boundary. Waves and currents are generated in the form of the water surface elevation and the horizontal and vertical orbital velocities described as a function of time. The data from the field tests will then be used as input data for the waves generated in the numerical model (Fig. 8). The measured water surface elevation (wave heights) and the deduced velocity field (development of horizontal and vertical velocity below the wave) are imported into the numerical model as time series for every time step. Hence, the measured wave is generated directly in the numerical simulation.

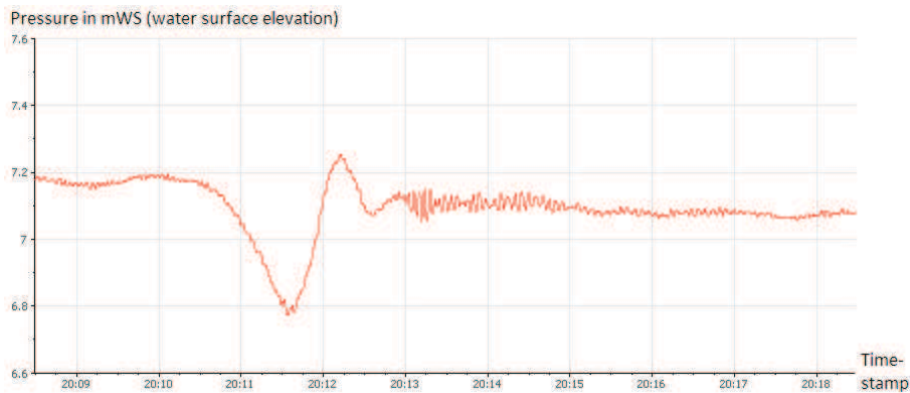


Figure 8: Example of measurement of ship-induced wave from field test.

The reaction of the rip-rap stones in the DEM code due to current and wave attack from the hydraulic calculation in CCFD can be recorded by using so called “histories” in PFC3D.

6 CFD-DEM coupling

The numerical codes used in this research project allow the two-way coupled computation of the mechanical DEM calculation and the hydraulic computation (see example Fig. 7). The displacement and the velocity of the particles are determined in PFC3D and the state variables of the flow in CCFD. During the coupled computation both programmes are executed using the procedure referred to above (section 4 and 5) with an additional data exchange at predefined time intervals. The DEM as well as the CFD code are formulated with additional terms to account for the fluid forces and the presence of particles in the flow.

In the DEM-part of the coupled computation a new term \vec{f}_{fluid} is added to the calculation to represent the force applied by the fluid (ITASCA 2008b):

$$\frac{\partial \vec{u}}{\partial t} = \frac{\vec{f}_{mech} + \vec{f}_{fluid}}{m} + \vec{g} \quad (3)$$

where \vec{u} is the particle velocity, m is the particle mass, \vec{f}_{fluid} is the total force applied by the fluid on the particle, \vec{f}_{mech} is the sum of additional forces and \vec{g} the acceleration of gravity. The force \vec{f}_{fluid} consists of three terms: the drag force, the force applied by the fluid pressure gradient and the buoyancy (ITASCA 2008b):

$$\vec{f}_{fluid} = \vec{f}_{drag} + \frac{4}{3} \pi r^3 (\nabla p - \rho \vec{g}) \quad (4)$$

with

$$\vec{f}_{drag} = \left(\frac{1}{2} C_d \rho \pi r^2 |\vec{u} - \vec{v}| (\vec{u} - \vec{v}) \right) \cdot n^{-\kappa} \quad (5)$$

where r is the particle radius, C_d is a drag coefficient and $n^{-\kappa}$ is an empirical factor to account for the local porosity. The current fluid velocity \vec{v} and the fluid pressure gradient ∇p necessary for this calculation are determined by CCFD and sent to PFC3D each time the coupling information are exchanged. The fluid force acting on the particles is applied to each particle (ITASCA 2008b).

In the hydraulic part of the computation the equations of motion of the fluid (1) and the continuity equation (2) are formulated with porosity terms and an additional body force due to the presence of particles in the flow (ITASCA 2008b):

$$\rho \frac{\partial n \vec{v}}{\partial t} + \rho \vec{v} \cdot \nabla (n \vec{v}) = -n \nabla p + \eta \nabla^2 (n \vec{v}) + \vec{f}_b \quad (6)$$

and

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \vec{v}) = 0 \quad (7)$$

with n as the porosity and \vec{f}_b as the body force. The current porosity n in each fluid element and the body force \vec{f}_b is determined in PFC3D and sent to CCFD during the data exchange. The body force acting on the fluid due to the particles is given as an average value over one fluid element (ITASCA 2008b).

The calculation cycle described above is shown in Fig. 9.

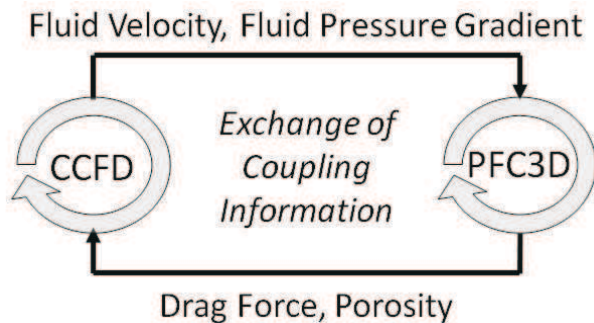


Figure 9: Calculation cycle of coupled DEM-CFD computation.

A limiting condition for the coupled computation is the minimum porosity of 1 % in each fluid element, in order to allow the flow of fluid. This means that a maximum of 99 % of the volume of one fluid element can be filled with particles. This affects the mesh resolution of the flow problem and results in fluid elements larger than the generated rip-rap stones.

7 Armourstone equipped with sensor technology

In the context of the research project a measurement device is developed to record the translational and rotational movements of armourstones due to current and wave attack. Several armourstones of different size, shape and density are bored and equipped with acceleration sensors and gyroscopes (Fig. 10). The corresponding circuit board for measuring acceleration and rotational speed is an in-house development. In this context the particular challenge was the limited space available, especially for the internal energy supply, as well as the minimisation of the electricity consumption of the system. Data can be registered by the stone for a period of up to one month and stored on a memory card. The displacements of the stone measured by the equipment in model or field tests can be compared to the displacements and the translational and angular velocities taken as histories from the particles in the numerical model. It is intended to determine the hydraulic loads and forces acting on the rip-rap from these measurements. Measurements with the equipped stones are carried out in the flume tests as well as in the field tests.

Fig. 11 shows the result of an equipped stone in the hydraulic flume embedded loosely in a rip-rap section during constant overflow with increasing flow velocity. The development of the angular velocity of roll, pitch and yaw angle during the test is shown. There is not stone movement at the beginning of the stepwise increase in flow velocity, but the stone starts shaking at a flow velocity of about 1.7 m/s to 2.0 m/s.



Figure 10: Example of armourstone equipped with sensor technology.

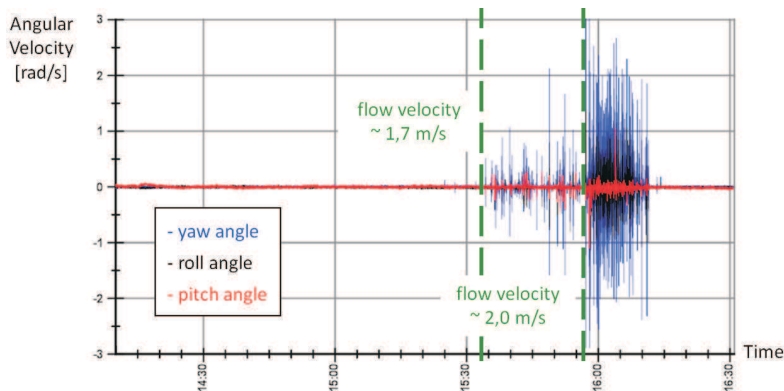


Figure 11: Example of measurement result from flume test.

8 Physical model tests

Physical model tests are used to validate the numerical model. The behaviour of the numerical armourstones depends primarily on the particle shape and the friction coefficient of the particle surface. Because a detailed representation of the stones raises the calculation time enormously, flume tests are performed to examine the extent to which these properties affect the accuracy of the numerical modelling and how the parameters should be chosen to achieve the best possible match with reality. The results of the physical tests with known boundary conditions are compared to an equivalent numerical model (Fig. 12).

The physical model tests are carried out in a hydraulic flume of the Federal Waterways Engineering and Research Institute, Hamburg. A rip-rap section on a scale of 1:1 with stones of the armourstone grading category CP_{90/250} is built into the flume (Fig. 12). There is an overflow parallel to the slope of the rip-rap section. The slope ratio of the rip-rap section is 1:1 respectively 1:3 in two different measurement campaigns. The flow in the zone of the rip-rap section is increased in four steps from about 1 m/s up to 2 m/s in several tests during one measurement campaign. The displacement of the stones due to the overflow is documented by a laser scan and a coloured surface of the rip-rap section.

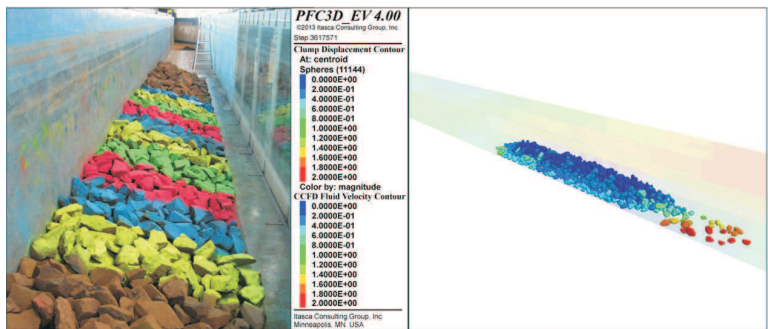


Figure 12: Rip-rap section in hydraulic flume and numerical simulation of physical model tests.

9 Field Tests

In addition to the physical model tests the numerical model is validated by field measurements executed as part of a project. Wave heights, flow velocities and to some extent pore water pressures in the subsoil are measured at the island “Lühesand” in the Lower Elbe River. The measurements are carried out at two different monitoring stations in exposed positions with varying slope ratios and varying distances to the navigation channel of the Elbe (Fig. 13). Stationary measurement systems and two flexible flow probes with independent power supply are applied. This equipment is placed at different positions over the entire slope of the bank and measures the acting hydraulic loads directly above the surface of the slope. Together with the measurement of the hydraulic loads, measurements with equipped armourstones are carried out as well.

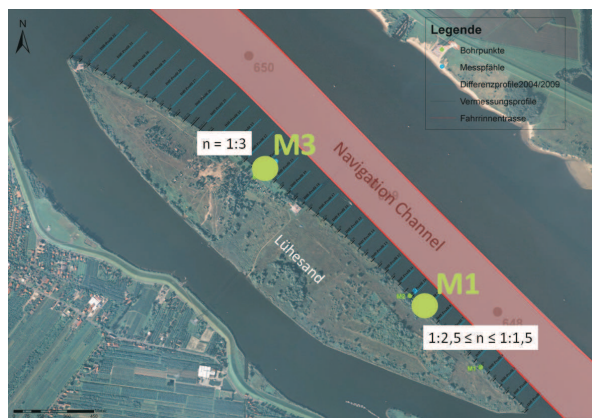


Figure 13: Monitoring positions M1 and M3 at island Lühesand, Elbe, Germany.

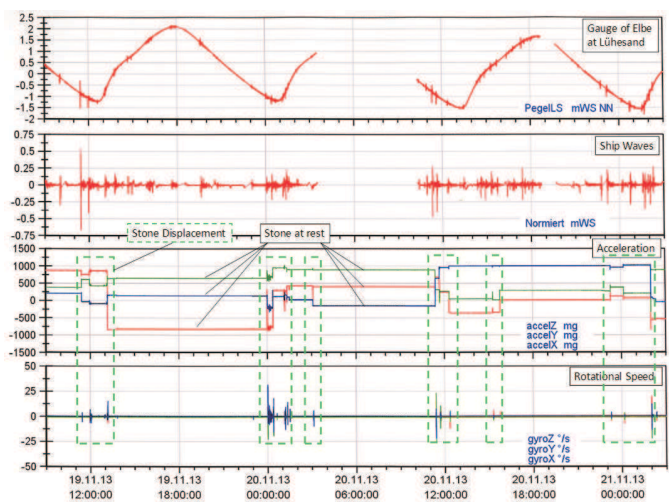


Figure 14: Example of measurements with equipped armourstones in field test.

Fig. 14 shows the measurement results with the equipped armourstone in the field tests over a period of 42 hours. From top to bottom the figure shows the tidally varying water level of the Elbe (gauge at Lühesand), the ship passages respectively the occurring ship waves, the acceleration in x-, y- and z-direction and the rotational speed in x-, y- and z-direction. This example taken from the first measurements demonstrates that displacements of single rip-rap stones mainly take place when ships are passing in periods of low tide.

10 Conclusion and outlook

The Discrete Element Method is a suitable method for the simulation of rip-raps with complex-shaped particles. The movement of single stones can be reproduced with any number of degrees of freedom. Together with a CFD-computation, the interaction of rip-rap and hydraulic loads is modelled holistically. The numerical particles representing rip-rap stones are calibrated using physical tests in a hydraulic flume and a measurement device for recording the movement of stones. The physical model tests and field tests will be continued and the modelling and validation of the numerical simulation will be adapted and improved to achieve the best possible match with reality.

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